NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3084

A METHOD FOR MEASURING THE PRODUCT OF INERTIA AND THE INCLINATION OF THE PRINCIPAL LONGITUDINAL AXIS OF INERTIA OF AN AIRPLANE

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SUMMARY

A description of a simple method of experimentally determining the product of inertia and the inclination of the principal longitudinal axis of inertia of an airplane is presented. The results of the application of this method and a description of the associated equipment and techniques are given for both a simple model and a conventional airplane. Previously reported methods for the determination of the moments of inertia about the pitch and roll body axes are reviewed for use in the determination of the angle of inclination of the principal longitudinal axis.

The angle of inclination of the principal axis was found to have a probable error of $\pm 0.17^{\circ}$ both from analysis of the error of inertia measurement of the full-scale airplane and from tests with a model having a simple configuration. Analysis showed the probable error of the inertia measurements for the test airplane to be ±1.00, ±0.49, and ±0.35 percent of the true moment of inertia about the X, Y, and Z body reference axes, respectively.

INTRODUCTION

Previous investigations have shown that the lateral dynamic stability characteristics of high-speed airplanes are strongly dependent on the moments of inertia and particularly on the product of inertia or the direction of the principal longitudinal axis of inertia (ref. 1). Accurate values of these quantities are required either to compute the response characteristics of an airplane from a knowledge of its stability derivatives or to determine the stability derivatives from measured response characteristics. Experience has indicated that accurate calculated values of moments of inertia and products of inertia based on the mass distribution of the airplane are not usually available. For this reason, direct measurement of these quantities is necessary.

The compound and bifilar pendulum methods for obtaining the moments and product of inertia are described in references 2 and 3. These methods require the construction of a special cradle for hoisting the airplane, and accurate determination of the direction of the principal axis requires suspending the airplane with its longitudinal axis tilted at a large angle with respect to the horizontal. For a large or heavy airplane, the determination of the direction of the principal axis becomes difficult and dangerous because, inasmuch as the airplane is tilted at a large angle, extensive damage would result if it were dropped. Methods have been described in reference 4 for determining the moments of inertia of an airplane by supporting it on pivots and oscillating it while it is restrained by springs. Similar methods, in conjunction with a method for determining the product of inertia, are described in reference 5. In addition, a method for determining the direction of the principal axis by suspending the airplane on a torsion rod is presented in reference 5. The use of this method avoided the necessity of tilting the airplane to large angles but it did require the construction of a suitable cradle and torsion rod.

In the present report, the product of inertia and the moment of inertia about the yaw body axis are obtained by suspending the airplane at a level attitude by means of a hoisting sling. The only auxiliary equipment required consists of suitable springs and the brackets for attaching them to the airplane. This method was tested by determining the inclination of the principal longitudinal axis of a small model of simple geometric shape. The method was then used to determine the moments of inertia and the product of inertia of a fighter airplane. The procedure used and the results obtained are described in this report.

In obtaining the measurements, flexibility of the wing of the airplane was found to have an appreciable effect on the determination of the moment of inertia about the longitudinal axis. A brief discussion of the method used for estimating this effect is also included.

SYMBOLS

c_X, c_Y — static spring constants of restraining springs for X- and Y-axes of oscillation, respectively, lb/ft

c_Z — equivalent spring constant of yaw restraining springs and spring attaching plates, ft-lb/radian

$\bar{c}$ — mean aerodynamic chord, ft

g — acceleration due to gravity, 32.2 ft/sec^2

h_X, h_Y	vertical components of distance from X- and Y-axes of oscillation, respectively, to airplane center of gravity, ft
I_X, I_Y, I_Z	moments of inertia about roll, pitch, and yaw axes, respectively (the axes are further defined by subscripts), slug-ft^2
ΔI	moment of inertia of sections of airplane, slug-ft^2
I_{XZ}	product of inertia, slug-ft^2
K	correction for wing flexibility (further defined by subscripts)
L	moment about roll axis, ft-lb
l_X, l_Y	perpendicular distance from center line of restraining spring to X- and Y-axes of oscillation, respectively, ft
l_X', l_Y'	perpendicular distance from X- and Y-axes of oscillation, respectively, to airplane center of gravity, ft
M	moment about principal longitudinal axis, ft-lb
N	moment about yaw axis, ft-lb
P_X, P_Y, P_Z	period of oscillation about X-, Y-, and Z-axes of oscillation, respectively, sec
V	total volume of airplane, cu ft
W	airplane weight, lb
α	angular acceleration about principal longitudinal axis, radians/sec^2
δ	angle in XZ-plane between X body reference axis and line connecting forward and rearward yaw spring attachment points (positive direction shown in fig. 1), deg
ϵ	angle in XZ-plane between X body reference axis and principal X-axis (positive direction shown in fig. 1), deg
ρ	air density at altitude of measurements, slugs/cu ft

φ angular displacement in roll, deg

ψ angular displacement in yaw, deg

Subscripts:

a additional mass

c spring-deflection correction

e moment-of-inertia equipment attached to facilitate measurements

f fuselage

m as measured (uncorrected for equipment, additional mass, etc.)

p principal axis passing through airplane center of gravity

r body reference axis passing through airplane center of gravity

s change due to shift in axis and entrapped air

se section

sp spring

α weighted mean of section acceleration corrections

1 load condition 1 (no fuel)

2 load condition 2 (full fuel)

A double dot over a symbol indicates a second derivative with respect to time.

DESCRIPTION OF METHOD

Determination of product of inertia and direction of principal axis.- In order to determine the product of inertia I_{XZ_r} or the angle of inclination ϵ by the method presented herein, the airplane is suspended by means of a single cable attached to a hoisting sling. The airplane may be placed at any desired attitude in pitch, but usually a near-level attitude is most convenient. The airplane is restrained in yaw by two pairs of springs attached to the fuselage ahead of and behind the center of gravity. A sketch which shows the points of attachment of the springs and defines some of the symbols and axes used in the derivation is presented in figure 1. The springs which are attached to brackets allow variation of δ, the angle between the horizontal and the plane through the points of attachment of the two sets of springs. The relative stiffness and location of the two sets of springs should be such that a rotation of the airplane in yaw about the hoisting cable produces no side force. The airplane is oscillated in yaw at a given attitude in pitch with various positions of the springs so that the angle δ is varied. In general, some coupling between yaw and roll is present, with the result that a certain amount of rolling motion occurs when the airplane is oscillated in yaw. At some particular value of the angle δ, however, the rolling motion accompanying the yawing oscillation is zero. The inclination of the principal axis from the horizontal may then be related to this angle δ at which the rolling motion is zero by means of a simple formula obtained from the following derivation.

The horizontal and vertical axes passing through the airplane center of gravity when the airplane is in a suspended position are defined as the reference axes for purposes of this discussion. The vertical plane passing through the center line of the airplane is assumed to be a plane of symmetry and therefore the pitch axis is a principal axis of inertia. Then, the equations of motion in roll and yaw are

Equation in pitch $M = I_{XZr}(\dot{\psi}^2 - \dot{\phi}^2)$ neglected.

$$I_{X_r} \ddot{\phi} - I_{XZ_r} \ddot{\psi} = L \quad \text{(Roll)} \tag{1a}$$

$$-I_{XZ_r} \ddot{\phi} + I_{Z_r} \ddot{\psi} = N \quad \text{(yaw)} \tag{1b}$$

If the angle δ is adjusted to compensate for the coupling between yaw and roll and there is pure yawing oscillation about the Z reference axis, then

$$-I_{XZ_r} \ddot{\psi} = L$$

$$I_{Z_r} \ddot{\psi} = N$$

or

$$I_{XZ_r} = -I_{Z_r} \frac{L}{N}$$

The moment produced by the springs may be represented by a vector perpendicular to the plane in which the points of attachment of the springs lie. If this resultant moment is resolved along the X and Z reference axes, the spring must have a component restoring moment N to give the yawing oscillation about the Z reference axis and must also have a component moment $-L$ along the X reference axis. Then,

$$\tan \delta = -\frac{L}{N}$$

Hence,

$$I_{XZ_r} = I_{Z_r} \tan \delta \tag{2}$$

The relation for the inclination of the principal axis of inertia in terms of the product of inertia is

$$\tan 2\epsilon = \frac{2I_{XZ_r}}{I_{Z_r} - I_{X_r}}$$

Solving for ϵ gives

$$\epsilon = \frac{1}{2} \tan^{-1} \frac{2I_{XZ_r}}{I_{Z_r} - I_{X_r}} \tag{3}$$

or ϵ in terms of δ (from eq. (2)) is

$$\epsilon = \frac{1}{2} \tan^{-1} \frac{2I_{Z_r} \tan \delta}{I_{Z_r} - I_{X_r}}$$

The angle ϵ is therefore not (as might have been expected) equal to δ but is related to it by a relation involving the moments of inertia about the X and Z reference axes.

In practice, the moment of inertia about the Z reference axis may be determined from the same tests used to determine the product of inertia. Determination of the moment of inertia about the X reference axis (see eq. (3)) requires a different setup (such as that described in ref. 5) in which the airplane is placed on pivots to provide freedom in roll and is restrained by springs.

Determination of principal moments of inertia.- The measured values of moments of inertia about the reference axes and the measured inclination of the principal axes may be used to determine the principal moments of inertia from the relations derived as follows.

Since the pitch axis is perpendicular to the plane of symmetry,

$$I_{Y_p} = I_{Y_r} \tag{4}$$

If the airplane rotates about the principal longitudinal axis with an angular acceleration α and a resulting moment M, then

$$I_{X_p} \alpha = M$$

Both the angular acceleration and the moment vectors may be resolved along the X and Z reference axes. The components along the X reference axis are related by the equation

$$I_{X_p} \ddot{\phi} = L \tag{5}$$

This value of L must also satisfy equation (1a) which applies for any type of motion. If equation (5) is equated to equation (1a) and the results are simplified, the following expression is obtained:

$$\left(I_{X_r} - I_{X_p}\right) \ddot{\phi} - I_{XZ_r} \ddot{\psi} = 0 \tag{6}$$

Dividing equation (6) by $\ddot{\phi}$ and solving for I_{X_p} gives

$$I_{X_p} = I_{X_r} - I_{XZ_r} \frac{\ddot{\psi}}{\ddot{\phi}}$$

or

$$I_{X_p} = I_{X_r} - I_{XZ_r} \tan \epsilon \tag{7}$$

where $\ddot{\psi}/\ddot{\phi} = \tan \epsilon$ for motion about the X principal axis.

The moment of inertia about the Z principal axis can be found in a similar manner by assuming that the airplane rotates about the Z principal axis. The resulting equation is

$$I_{Z_p} = I_{Z_r} + I_{XZ_r} \tan \epsilon \tag{8}$$

Equations (7) and (8) are equivalent to the standard inertia equations

$$I_{X_p} = I_{X_r} \cos^2\epsilon + I_{Z_r} \sin^2\epsilon - 2I_{XZ_r} \sin \epsilon \cos \epsilon$$

and

$$I_{Z_p} = I_{X_r} \sin^2\epsilon + I_{Z_r} \cos^2\epsilon + 2I_{XZ_r} \sin \epsilon \cos \epsilon$$

which are given in most engineering handbooks and are derived in reference 5. For numerical calculation, however, equations (7) and (8) are more convenient.

MODEL TESTS

In order to determine whether the method just described would give sufficiently accurate measurements of the inclination of the principal axis, tests were made with a small model of simple geometric shape for which the moments and product of inertia could be accurately calculated. Because the nature of the rolling motion of an airplane suspended from a single cable attached to a hoisting sling is fairly complex, some experimentation was considered desirable to determine the most suitable arrangement of the springs and suspension system. Another object of the tests was to determine whether aerodynamic forces on the vertical tail would appreciably influence the determination of the inclination of the principal axis.

A photograph of the test setup used in these experiments is shown in figure 2. The inertia of the airplane was represented by a pair of steel bars in the form of a cross. Tests were made with two sets of restraining springs, one designated as light springs and the other designated as heavy springs. The suspension system shown in figure 2 was used for one set of tests; for a second set of tests, the point of attachment of the three wires simulating the hoisting sling was restrained against lateral motion. Tests were also made with a balsa-wood fin (sufficiently large to account for aerodynamic effects) simulating the vertical tail of an airplane. Additional tests were made with added mass located unsymmetrically, as shown in figure 2, to produce a known tilt of the principal axis.

When the model is suspended as shown in figure 2, two modes of rolling motion exist. These modes, shown in figure 3, are designated the rolling mode and the modified pendulum mode. When the sling attachment point is fixed, only a pendulum mode of motion is possible. Table I summarizes the periods of the various modes that were obtained with the model in each test condition before the addition of the unsymmetrical masses. With the spring attachment points located below the center of gravity, very little rolling of the model was found to occur in the modified pendulum mode, a condition which is desirable because very little response in this mode then occurs during a yawing oscillation.

The spring position which would result in zero rolling motion was determined by plotting the ratio of the maximum roll amplitude to the maximum yaw amplitude as a function of the tangent of the spring angle δ. Plots of this type are shown in figure 4. This figure shows that the use of the heavy springs gave the greatest response in roll for a given value of the spring angle. This response is thought to result from the fact that the period of the rolling mode was close to that of the yawing mode so that the response in roll to the rolling moments resulting from the yawing oscillation was near a resonant condition. Actually, a smooth-wave form of the rolling motion was difficult to attain under these conditions and some unsteady rolling motion of the symmetrical model generally occurred even at the point of zero spring inclination. With the light springs, the rolling motion was appreciably reduced, but the wave form of the rolling motion was a smooth sine wave so that the accuracy in determining the point of zero rolling motion was limited mainly by the error in reading the film. For this reason approximately equal accuracy was obtained with either set of springs. When the sling attachment point was fixed, the amplitude of the rolling motion was still further reduced and this reduction would probably result in some additional loss of accuracy.

The principal axis of the model was then inclined by the addition of weights as shown in figure 2. This inclination gave the model a calculated product of inertia about the XZ centroidal axes of 0.00772 slug-ft^2. The calculated values for the moments of inertia about the X and Z centroidal axes were 0.0571 and 0.109 slug-ft^2, respectively. These values correspond to an angle of inclination of the principal axis of 8.31°. The results of tests to determine the spring angle for zero rolling are shown in figure 5. From figure 5, tangent δ has a value of 0.072 when the ratio of roll amplitude to yaw amplitude is zero. By using this experimental value together with the calculated moments of inertia, the angle of inclination of the principal axis was found to be 8.46° which is in good agreement with the calculated value of 8.31°.

The conclusions obtained from these tests are: (1) The presence of the two modes of rolling motion did not create any difficulty but the period of the rolling mode should not be too close to the period of the yawing oscillation, (2) no advantage was gained by fixing the sling attachment point, (3) adding the simulated vertical fin, which had a negligible mass effect on the calculated inclination of the principal axis, had no measurable effect on the experimentally determined value.

FULL-SCALE TESTS

After the method of determining the inclination of the principal axis had been shown to be feasible on the basis of model tests, the method was tried on a full-scale fighter airplane. The moments of inertia about the X, Y, and Z body reference axes of the airplane were required for the calculation of the product of inertia, the inclination of the principal axis, and the principal moments of inertia. A method similar to that described in reference 5 was used for measuring the moments of inertia about the X and Y body reference axes.

Determination of position of airplane center of gravity.- The longitudinal position of the center of gravity for load conditions 1 (no fuel) and 2 (full fuel) was determined by weighing the airplane on electric scales. The vertical position of the center of gravity for load condition 1 was determined by the plumb-line suspension method given in reference 2 and the vertical position of the center of gravity for load condition 2 was determined by calculating the amount the center of gravity would move from load condition 1 because of the weight and position of the added fuel.

Determination of moment of inertia about X reference axis.- A photograph of the airplane and equipment for measuring the moment of inertia about the roll axis is shown as figure 6. The instrumentation consisted of a synchro transmitter and recorder coupled to a 1/10-second timer. The transmitter was positioned near the right wing tip with an arm attached to the transmitter shaft and resting on a straightedge parallel to the X reference axis and fastened to the wing tip. The transmitter thus recorded the period of manually induced rolling oscillations.

The observed moment of inertia of the airplane and equipment about the roll axis of oscillation is given by

$$I_{X_m} = \left(c_X l_X^2 - Wh_X\right)\left(\frac{P_X}{2\pi}\right)^2$$

The moment of inertia about the roll axis through the airplane center of gravity and parallel to the axis of oscillation is given by the equation

$$I_{X_r} = I_{X_m} - I_{X_e} - I_{X_a} - I_{X_s} \qquad (9)$$

where I_{X_e} is the moment of inertia of the inertia test equipment about the axis of oscillation, I_{X_a} is the moment of inertia due to the apparent additional-mass effect of oscillating in a fluid medium, and I_{X_s} is equivalent to $\left(\frac{W}{g} + V\rho\right)l_X'^2$ which represents the correction for transfer of axes and takes into account the entrapped air and the buoyancy effect of the displaced air at the altitude of the ground measurements.

Table II gives the corrections considered for the equipment used in measuring the moments of inertia, reference 6 presents formulas and graphs to find additional mass corrections, and reference 2 explains corrections for inertia due to the combined effects of the buoyancy of the surrounding medium and the shifting of axes.

During the full-scale measurements about the X reference axis it was found necessary to make corrections for wing flexibility. These corrections would be particularly important for configurations having thin wings of high aspect ratio. For the test airplane, this correction was found to be approximately 5 percent of the moment of inertia about the X reference axis.

When the airplane is at any roll angle during a sinusoidal rolling oscillation about the knife edges, the wings are flexed because of the resultant loading produced by distributed inertial loading and concentrated spring restraint which act in opposite directions. Each part of the airplane has the same period of oscillation but different angular accelerations and displacements; therefore, the inertia equation for an airplane

$$\sum \Delta I\, \ddot{\phi}_{se} = L = c_X l_X^2 \phi_{sp} - Wh_X \phi_f$$

must be corrected to the same roll angle and rolling acceleration before integration. The roll angle of the spring can be related to the roll angle of the fuselage by means of a constant K_c where $\phi_{sp} = K_c \phi_f$ and, in a similar manner, the various rolling accelerations of the sections of the wing can be related to the rolling acceleration of the fuselage by means of a relation $\ddot{\phi}_{se} = K_{se}\ddot{\phi}_f$. The inertia equation for a flexible airplane would be

$$\left(\sum \Delta I \; K_{se}\right) \ddot{\phi}_f = \left(K_c c_X l_X{}^2 - Wh_X\right) \phi_f$$

or in the integrated form

$$\sum \Delta I \; K_{se} = \left(K_c c_X l_X{}^2 - Wh_X\right)\left(\frac{P_X}{2\pi}\right)^2$$

The value $\sum \Delta I_{se}$ can be related to $\sum \Delta I_{se} \; K_{se}$ by means of a constant K_α, so that

$$\sum \Delta I_{se} = K_\alpha \sum \Delta I_{se} \; K_{se}$$

or

$$\sum \Delta I_{se} = K_\alpha\left(K_c c_X l_X{}^2 - Wh_X\right)\left(\frac{P_X}{2\pi}\right)^2$$

Since the calculated value $\sum \Delta I$ is approximately equal to the measured. value I_{X_m} within the accuracy of the manufacturer's inertia-distribution data, then

$$I_{X_m} = K_\alpha\left(K_c c_X l_X{}^2 - Wh_X\right)\left(\frac{P_X}{2\pi}\right)^2$$

For the test airplane the values K_{se} and K_c were calculated from available static calibrations but, in general practice, it would be advisable to determine these constants as the ratio of the measured value of the maximum dynamic displacement of the sections involved and the calculated maximum displacement that the same section of the airplane would have were it part of a rigid airplane. The value of K_α can be approximately determined by means of calculated inertia-distribution data supplied by the manufacturer using the relation

$$K_\alpha = \frac{\sum \Delta I_{se}}{\sum \Delta I_{se} \; K_{se}}$$

Determination of moment of inertia about Y reference axis.- A knife-edge and restraining-spring system, similar in principle and handling procedure to that used to obtain measurements about the roll axis, was used to measure the moment of inertia about the pitch axis. The airplane was set up for tests as shown in figure 7. The same instrumentation used in the roll-axis measurements was attached to the tail of the airplane to obtain time histories of manually induced oscillations in pitch.

The moment of inertia about the pitch axis passing through the airplane center of gravity is given by the equation

$$I_{Y_r} = I_{Y_m} - I_{Y_e} - I_{Y_a} - I_{Y_s} \tag{10}$$

where

$$I_{Y_m} = \left(c_Y l_Y^2 - Wh_Y\right)\left(\frac{P_Y}{2\pi}\right)^2$$

and

$$I_{Y_s} = \left(\frac{W}{g} + V\rho\right) l_Y'^2$$

Determination of moment of inertia about Z reference axis and product of inertia.- The moment of inertia about the Z reference axis is determined from the same tests used to determine the product of inertia. The airplane as set up for the tests is shown in figure 8. The springs between the A-frames and the spring attaching plates provided the necessary restraining couple and allowed the direction of the restoring moment of the springs to be varied to compensate for the angular momentum causing coupling between yaw and roll. The instrumentation consisted of two synchro transmitters and recorders coupled to a 1/10-second timer. The yaw transmitter was positioned at the nose of the airplane and the roll transmitter was positioned at the right wing tip in the same manner as for the pure roll case. It is important that the straightedge be level in this case so that the yaw position will not affect the recording of rolling motion. Records were taken only after the manually induced yaw caused by pushing on each wing tip and the roll caused by coupling due to yawing motion had reached a steady state.

If an airplane is suspended from a hoist by a sling, the center of gravity of the suspended airplane is directly below the attaching point of the airplane sling and the crane hook. The resultant axis of oscillation passes through the airplane center of gravity. The moment of inertia about the axis of the supporting cable is given by the equation

$$I_{Z_m} = c_Z\left(\frac{P_Z}{2\pi}\right)^2 \qquad (11)$$

where c_Z is the equivalent spring constant (in ft-lb/radian) of the restraining springs, spring attaching plates, and the airplane fuselage.

Equation (11) is exact only when the Z-axis of oscillation is the resultant axis of the combined moment vectors of the restoring springs and the airplane angular acceleration. The effects of rolling on the period of the oscillations in yaw are negligible, however, for angles of less than 4° between the resultant vector and the principal axis of inertia, so that equation (11) gives a valid approximation through that range.

The moment of inertia about the yaw reference axis passing through the airplane center of gravity is given by the equation

$$I_{Z_r} = I_{Z_m} - I_{Z_a} - I_{Z_e} \qquad (12)$$

Periods of the various modes of rolling motion of the airplane are given in table I. Typical time histories of yawing and rolling oscillations are given in figure 9. Plots of the ratio of maximum roll amplitude to maximum yaw amplitude against the tangent of the spring angle for load conditions 1 and 2 are given in figures 10 and 11, respectively. The product of inertia and the inclination of the principal axis were determined using equations (2) and (3). The principal moments of inertia were determined using equations (4), (7), and (8). Sample calculations are given in the appendix using values presented in table III.

RESULTS AND DISCUSSION

Moments of Inertia About the Body Reference Axes

Basic data.- The dimensions and physical characteristics of the airplane are given in table III. Values for quantities which varied because of the inaccuracies of the equipment and methods used were obtained by averaging several measurements. Corrections were made, in the rolling case, for the flexibility of the airplane wing and, in the yawing case, for the flexibility of the spring attaching plates. Other corrections were made for the inertia equipment (see table II) and for the effects of the ambient air mass according to reference 6. The values of the moments of inertia about the body reference axes for load conditions 1 and 2 are given in table IV.

Precision.- The possible errors in the measured and computed quantities used in calculating the moment of inertia are summarized in table V. These possible errors in the variables were estimated on the basis of present equipment, test techniques, and previous experience (refs. 4 to 6). The probable error $\left(\text{based on the relation, Probable error} = 0.675\sqrt{\sum(\text{Possible errors})^2}\right)$ is a measure of the overall precision of the method.

Product of Inertia and Inclination of the Principal Axis

The values of I_{XZ_r} and ϵ are given in tables IV and VI, respectively, for load conditions 1 and 2. With regard to precision, the net effect on I_{XZ_r} and ϵ of errors in I_{X_r} and I_{Z_r} is small but the effect of δ, which had an approximately constant error of $\pm 0.1^\circ$ when determined for the point of zero roll for either the model or the airplane, is much more important. The resultant probable error for I_{XZ_r} is approximately $\pm 0.0018\ I_{Z_r}$ and the error for ϵ is approximately a constant value of $\pm 0.17^\circ$. The estimated error of I_{XZ_r} for the test airplane is ± 56 slug-ft^2.

Principal Moments of Inertia

The principal moments of inertia for the two load conditions are summarized in table VI. Since ϵ is small, the moments of inertia about the principal axes and the resulting probable errors are nearly the same as the moments of inertia about the body reference axes and their corresponding probable errors (see tables IV and V).

CONCLUDING REMARKS

The method employed in the present investigation for measuring the inclination of the principal longitudinal axis of inertia and the moments of inertia of an airplane reduced the handling problems and inherent inaccuracies of previous methods and appears suitable for application to inertia measurements on any airplane capable of being suspended on a hoisting sling.

The accuracy of the test method for the determination of the inclination of the principal axis was within $\pm 0.17^{\circ}$. The estimated probable error of the test airplane and the calculated error of the test model were of the same magnitude.

Tests with the full-scale airplane showed that it is important to consider the effects of the flexibility of the wings in determining the moment of inertia about the longitudinal axis.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 27, 1954.

APPENDIX

CALCULATION OF MOMENTS OF INERTIA, PRODUCT OF INERTIA, AND INCLINATION OF PRINCIPAL AXIS FOR LOAD CONDITION 1

The observed moments of inertia about the knife edges are:
About the X-axis,

$$I_{X_m} = K_\alpha\left(K_c c_X l_X^{\ 2} - Wh_X\right)\left(\frac{P_X}{2\pi}\right)^2$$

$$= 1.015\left[(0.947)(3,240)(17)^2 - (11,188)(3.025)\right]\left[\frac{0.8255}{2(3.1416)}\right]^2$$

$$= 14,942 \text{ slug-ft}^2$$

about the Y-axis,

$$I_{Y_m} = \left(c_Y l_Y^{\ 2} - Wh_Y\right)\left(\frac{P_Y}{2\pi}\right)^2$$

$$= \left[(4,850)(16.93)^2 - (11,188)(1.656)\right]\left[\frac{0.8955}{2(3.1416)}\right]^2$$

$$= 27,912 \text{ slug-ft}^2$$

and about the Z-axis,

$$I_{Z_m} = c_Z\left(\frac{P_Z}{2\pi}\right)^2$$

$$= 67,464\left[\frac{4.344}{2(3.1416)}\right]^2$$

$$= 32,247 \text{ slug-ft}^2$$

The moments of inertia about the body reference axes are:
About the X-axis,

$$I_{X_r} = I_{X_m} - I_{X_e} - I_{X_a} - I_{X_s}$$

$$= 14,942 - 414 - 388 - 3,194$$

$$= 10,946 \text{ slug-ft}^2$$

about the Y-axis,

$$I_{Y_r} = I_{Y_m} - I_{Y_e} - I_{Y_a} - I_{Y_s}$$

$$= 27,912 - 823 - 178 - 3,538$$

$$= 23,373 \text{ slug-ft}^2$$

and about the Z-axis,

$$I_{Z_r} = I_{Z_m} - I_{Z_e} - I_{Z_a}$$

$$= 32,247 - 190 - 187$$

$$= 31,870 \text{ slug-ft}^2$$

The product of inertia and the inclination of the principal axes are:

$$I_{XZ_r} = I_{Z_r} \tan \delta$$

$$= 31{,}870(0.00416)$$

$$= 133 \text{ slug-ft}^2$$

and

$$\epsilon = \frac{1}{2} \tan^{-1} \frac{2I_{XZ_r}}{I_{Z_r} - I_{X_r}}$$

$$= \frac{1}{2} \tan^{-1} \frac{2(133)}{31{,}870 - 10{,}946}$$

$$= 0.36^\circ$$

The principal moments of inertia are:
About the X-axis,

$$I_{X_p} = I_{X_r} - I_{XZ_r} \tan \epsilon$$

$$= 10{,}946 - (133)(0.00633)$$

$$= 10{,}945 \text{ slug-ft}^2$$

about the Y-axis,

$$I_{Y_p} = I_{Y_r}$$

$$= 23{,}373 \text{ slug-ft}^2$$

and about the Z-axis,

$$I_{Z_p} = I_{Z_r} + I_{XZ_r} \tan \epsilon$$

$$= 31{,}870 + (133)(0.00633)$$

$$= 31{,}871 \text{ slug-ft}^2$$

REFERENCES

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3. Gracey, William: The Experimental Determination of the Moments of Inertia of Airplanes by a Simplified Compound-Pendulum Method. NACA TN 1629, 1948.

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TABLE I

PERIODS OF VARIOUS MODES OF OSCILLATION OF

TEST MODEL AND TEST AIRPLANE

Condition		Period, sec		
Test unit	Sling attachment point	Yawing mode	Rolling mode	Pendulum mode
Model, light springs	Fixed	0.84	-----	0.42
	Free	.84	0.46	.40
Model, heavy springs	Fixed	.55	-----	.34
	Free	.55	.43	.32
Airplane	Free	4.344	1.698	2.265

TABLE II

MOMENTS OF INERTIA ABOUT KNIFE EDGES OF COMPONENT

PARTS OF INERTIA EQUIPMENT

[All units are measured in slug-ft^2.]

About X-axis:	
Nose support	0.18
I-beam and spacer supports	5.79
Roll springs (both)	139.11
Moving parts of both spring cages	179.50
Unit attaching spring cages to wings (both)	89.75
Total of inertia equipment for roll axis	414.33
About Y-axis:	
Pitch springs	209.17
Moving parts of spring cages	
Load condition 1	605.27
Load condition 2	338.24
Unit attaching spring cage to tail	8.90
Total of inertia equipment for pitch axis	
Load condition 1	823.34
Load condition 2	556.31
About Z-axis:	
Hoist fittings	83.89
Spring attaching plates	26.56
Springs (both pairs)	8.23
Spring attaching units for other axes	71.80
Total of inertia equipment for yaw axis	190.48

TABLE III

DIMENSIONS REQUIRED IN INERTIA MEASUREMENTS

Weight and balance:	
Load condition 1 (no fuel)	
Weight, lb	11,188
Longitudinal center-of-gravity position (gear up), percent $\bar{c}$	22.82
Vertical center-of-gravity position above fuselage reference line (gear up), ft	0.69
Load condition 2 (full fuel)	
Weight, lb	16,240
Longitudinal center-of-gravity position (gear up), percent $\bar{c}$	24.30
Vertical center-of-gravity position above fuselage reference line (gear up), ft	0.76
Dimensions of inertia measurements:	
About X-axis,	
Perpendicular distance from the axis of the spring to the axis of oscillation, l_X, ft	17
Spring constant of the restraining springs, c_X (total), lb/ft	3,240
Perpendicular distance from the axis of oscillation to the airplane center of gravity, l_X', ft	
Load condition 1	3.03
Load condition 2	3.09
Vertical component of the distance between the X-axis of oscillation and the airplane center of gravity, h_X, ft	
Load condition 1	3.03
Load condition 2	3.09
Correction for the dynamic flexing of the airplane wing at the spring attaching point, K_c	
Load condition 1	0.947
Load condition 2	0.944
Weighted mean correction of the section corrections to acceleration of the flexible airplane	
Load condition 1	1.015
Load condition 2	1.013
Period of oscillation, P_X, sec	
Load condition 1	0.8255
Load condition 2	0.9046
Moment of inertia of the inertia gear, I_{X_e}, slug-ft^2	414

TABLE III.- Continued

DIMENSIONS REQUIRED IN INERTIA MEASUREMENTS

Moment of inertia due to displacement of axis, I_{X_s}, slug-ft^2	
Load condition 1	3,194
Load condition 2	4,843
Moment of inertia due to ambient air, I_{X_a}, slug-ft^2	388
About Y-axis,	
Perpendicular distance from the axis of the spring to the axis of oscillation, l_Y, ft	16.93
Static spring constant of the restraining spring, c_Y, lb/ft	4,850
Perpendicular distance from the axis of oscillation to the airplane center of gravity, l'_Y, ft	
Load condition 1	3.18
Load condition 2	3.13
Vertical component of the distance between the Y-axis of oscillation and the airplane center of gravity, h_Y, ft	
Load condition 1	1.66
Load condition 2	1.73
Period of oscillation, P_Y, sec	
Load condition 1	0.8964
Load condition 2	0.9625
Moment of inertia of the inertia gear, I_{Y_e}, slug-ft^2	
Load condition 1	823
Load condition 2	556
Moment of inertia due to displacement of axis, I_{Y_s}, slug-ft^2	
Load condition 1	3,538
Load condition 2	4,953
Moment of inertia due to ambient air mass, I_{Y_a}, slug-ft^2	178
About Z-axis,	
Equivalent spring constant, c_Z, ft-lb/radian	67,464
Period of oscillation, P_Z, sec	
Load condition 1	4.344
Load condition 2	4.461

TABLE III.- Concluded

DIMENSIONS REQUIRED IN INERTIA MEASUREMENTS

Moment of inertia of inertia gear, I_{Z_e}, slug-ft^2	190
Moment of inertia due to ambient air mass, I_{Z_a}, slug-ft^2	
Load condition 1	187
Load condition 2	186
Tangent of the spring angle when only pure yaw exists, $\tan\delta$	
Load condition 1	0.004
Load condition 2	-0.016

TABLE IV

MOMENTS OF INERTIA AND PRODUCTS OF INERTIA

ABOUT BODY AXES THROUGH AIRPLANE

CENTER OF GRAVITY

	Load condition 1	Load condition 2
I_{X_r}, slug-ft^2	10,946	12,195
I_{Y_r}, slug-ft^2	23,373	26,282
I_{Z_r}, slug-ft^2	31,870	33,632
I_{XZ_r}, slug-ft^2	133	-527

TABLE V

PRECISION ANALYSIS OF TEST METHODS

Variables	Estimated error	Probable error, percent of true moment of inertia					
		Load condition 1			Load condition 2		
		I_{X_r}	I_{Y_r}	I_{Z_r}	I_{X_r}	I_{Y_r}	I_{Z_r}
c_X, c_Y, c_Z	±0.5 percent	±0.71	±0.61	±0.51	±0.75	±0.62	±0.51
$I_{X_a}, I_{Y_a}, I_{Z_a}$	±10 percent	±.35	±.08	±.06	±.32	±.07	±.05
$I_{X_e}, I_{Y_e}, I_{Z_e}$	±5 percent	±.19	±.18	±.03	±.17	±.11	±.03
I_{X_s}, I_{Y_s}	±1.5 percent	±.44	±.23	----	±.59	±.28	----
K_c	±0.5 percent	±.71	----	----	±.75	----	----
K_α	±0.5 percent	±.68	----	----	±.73	----	----
l	±0.01 foot	±.16	±.14	----	±.18	±.15	----
P	±0.05 percent	±.14	±.12	±.10	±.15	±.12	±.10
W	±10 pounds	$\ll \pm .01$	$\ll \pm .01$	----	$\ll \pm .01$	$\ll \pm .01$	----
h	±0.02 foot	±.03	±.02	----	±.05	±.03	----
Probable error		±0.92	±0.48	±0.35	±1.00	±0.49	±0.35

TABLE VI

PRINCIPAL MOMENTS OF INERTIA AND INCLINATION

OF PRINCIPAL AXIS

	Load condition 1	Load condition 2
I_{X_p}, slug-ft^2	10,945	12,182
I_{Y_p}, slug-ft^2	23,373	25,282
I_{Z_p}, slug-ft^2	31,871	33,645
ϵ, deg	0.36	-1.41

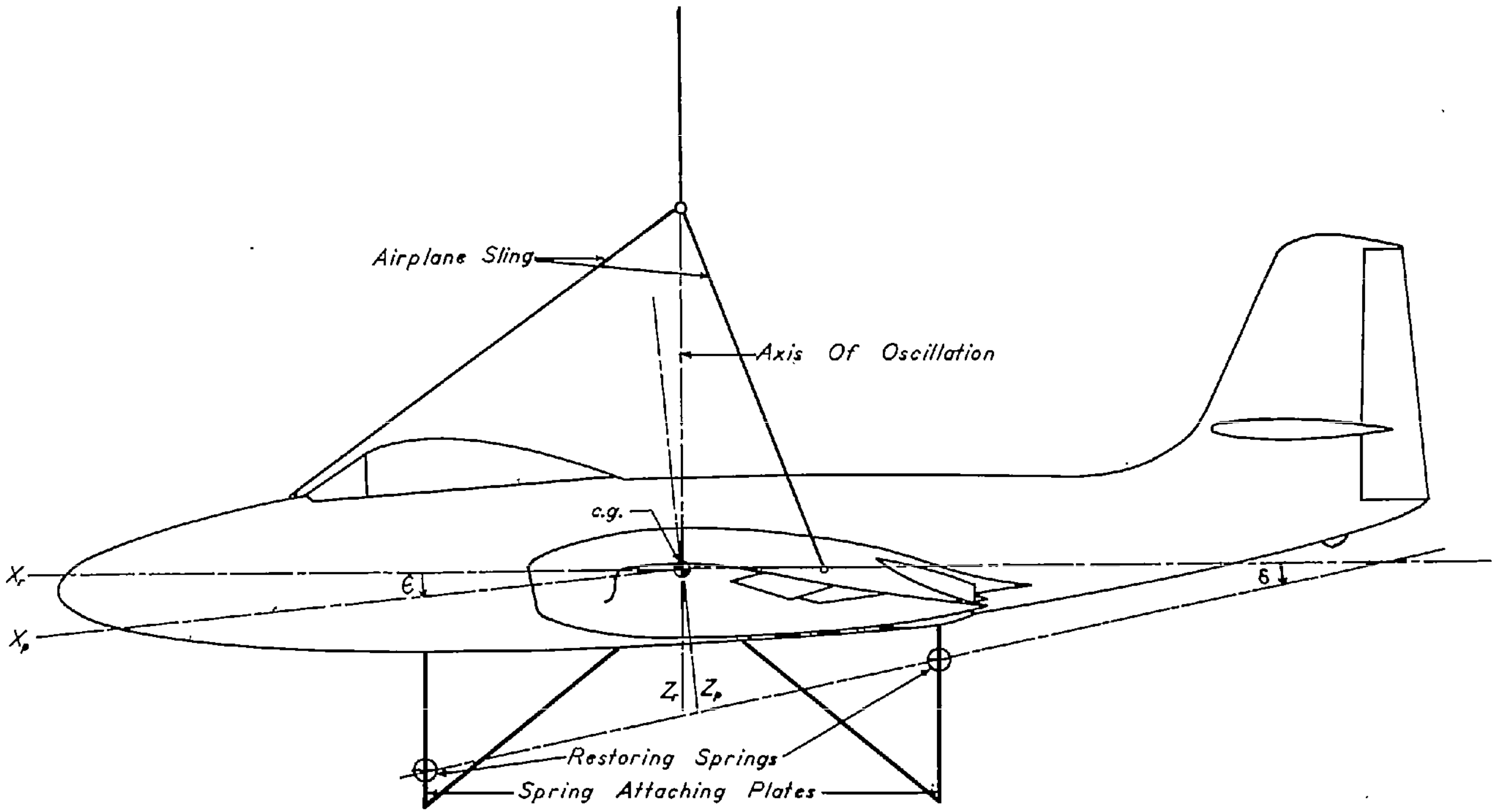

Figure 1.- Sketch of the test airplane showing pertinent symbols for the determination of principal axes.

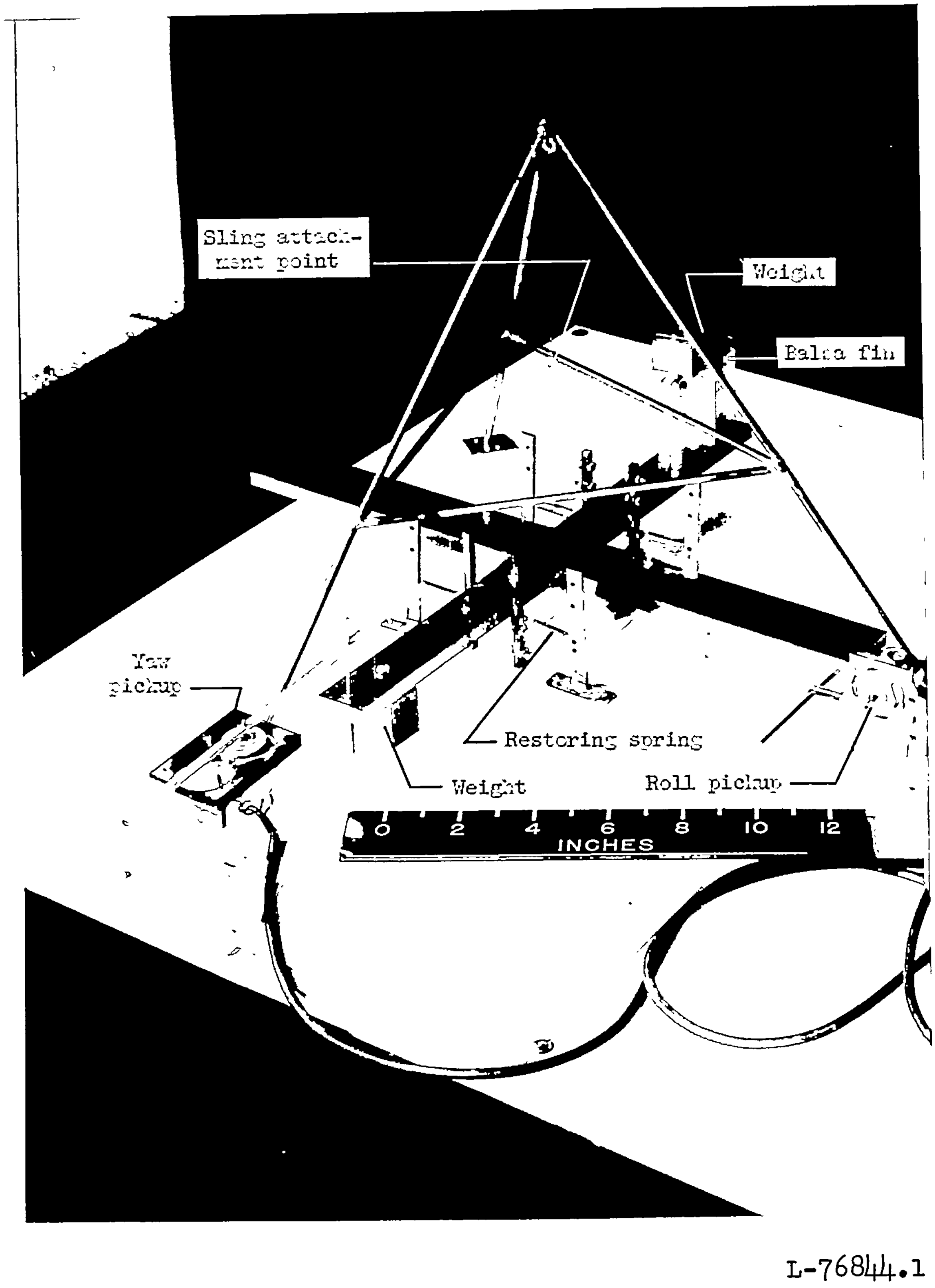

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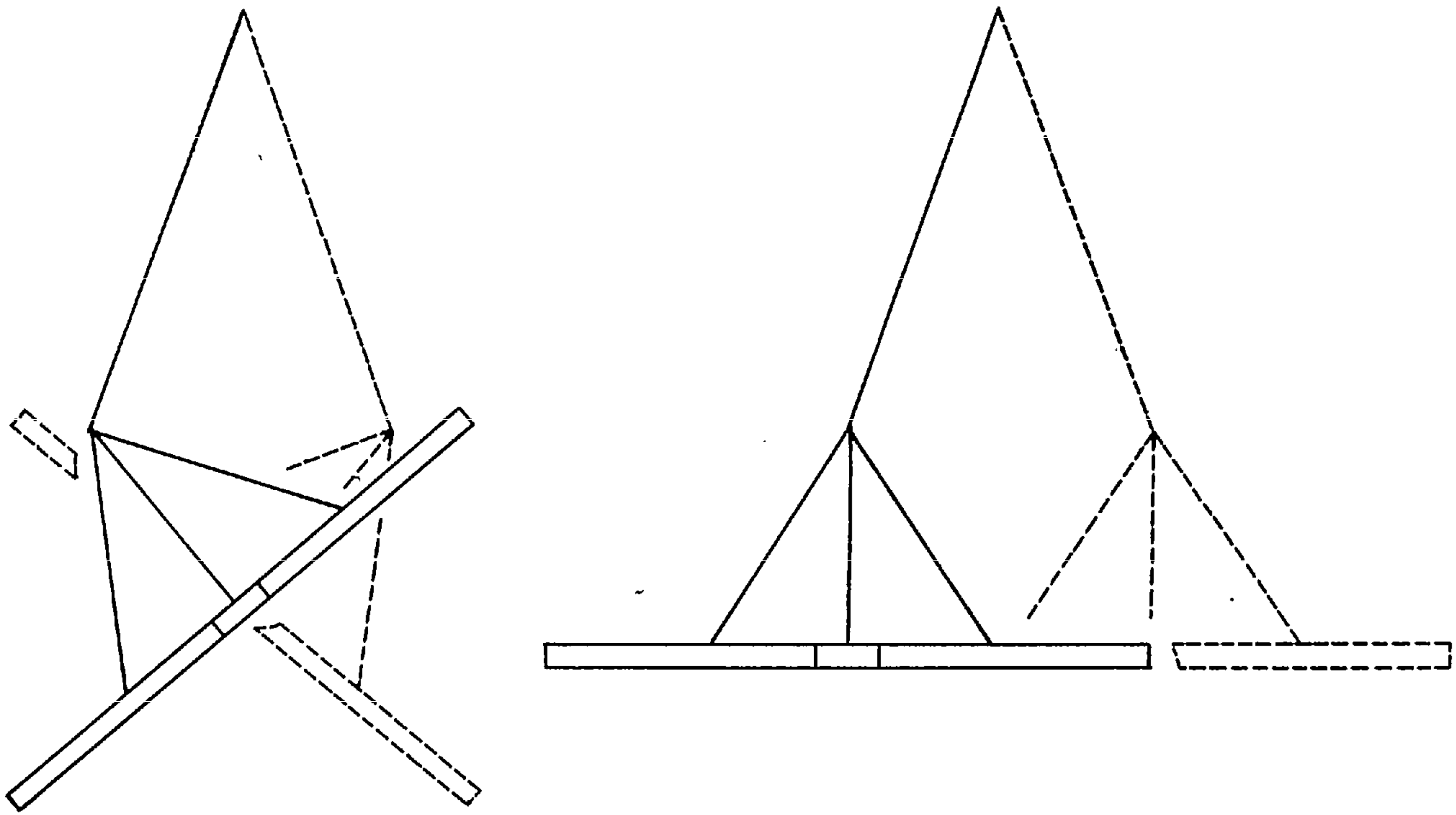
Figure 2.- Model with balsa fin and with principal axis inclined by weights.



(a) Rolling mode. (b) Pendulum mode.

Figure 3.- Modes of rolling motion of the test model as a result of the suspension system.

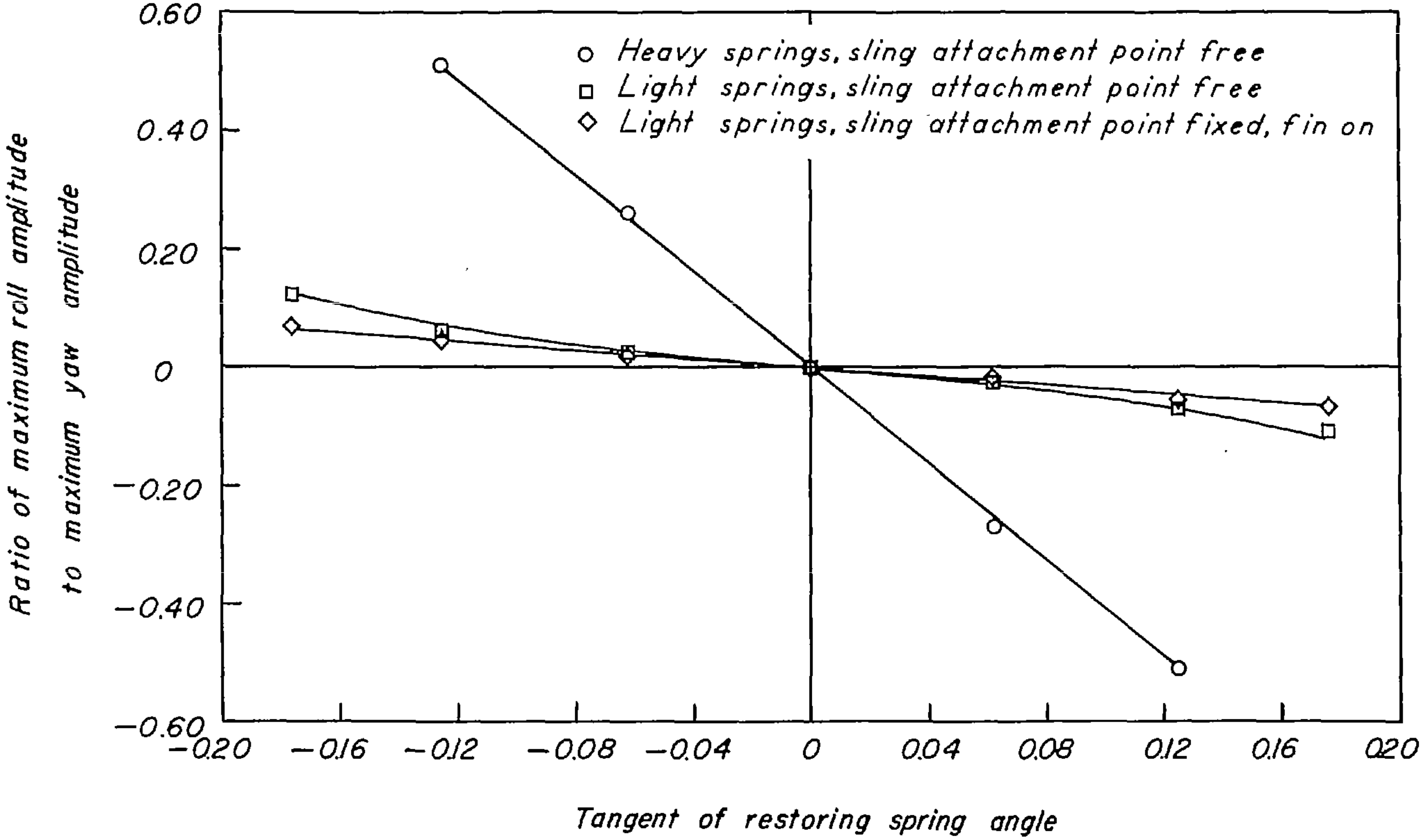


Figure 4.- Ratio of maximum roll amplitude to maximum yaw amplitude as a function of spring-restoring-moment direction for the symmetrical model.

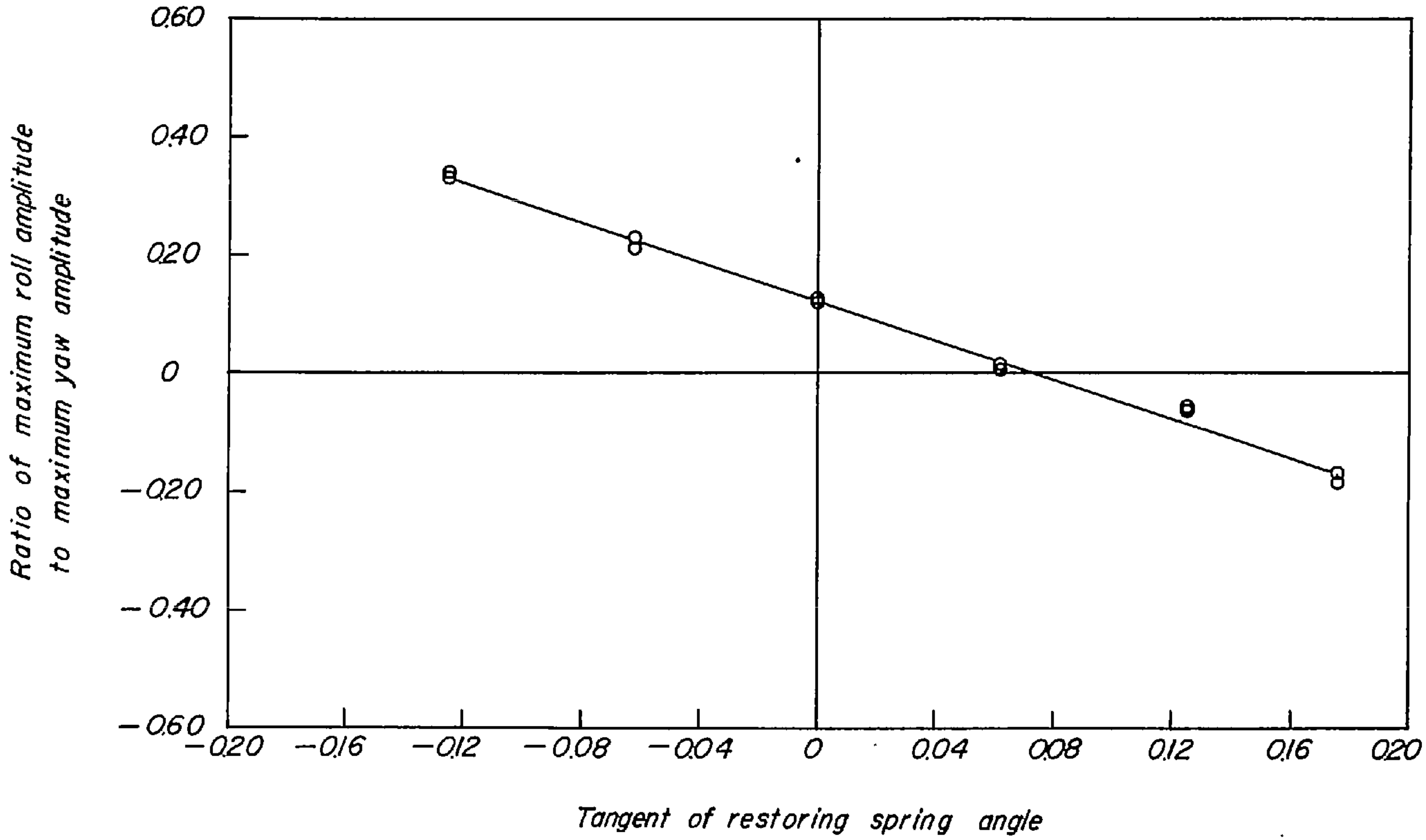


Figure 5.- Ratio of maximum roll amplitude to maximum yaw amplitude as a function of spring-restoring-moment direction for the model with the principal axis inclined.

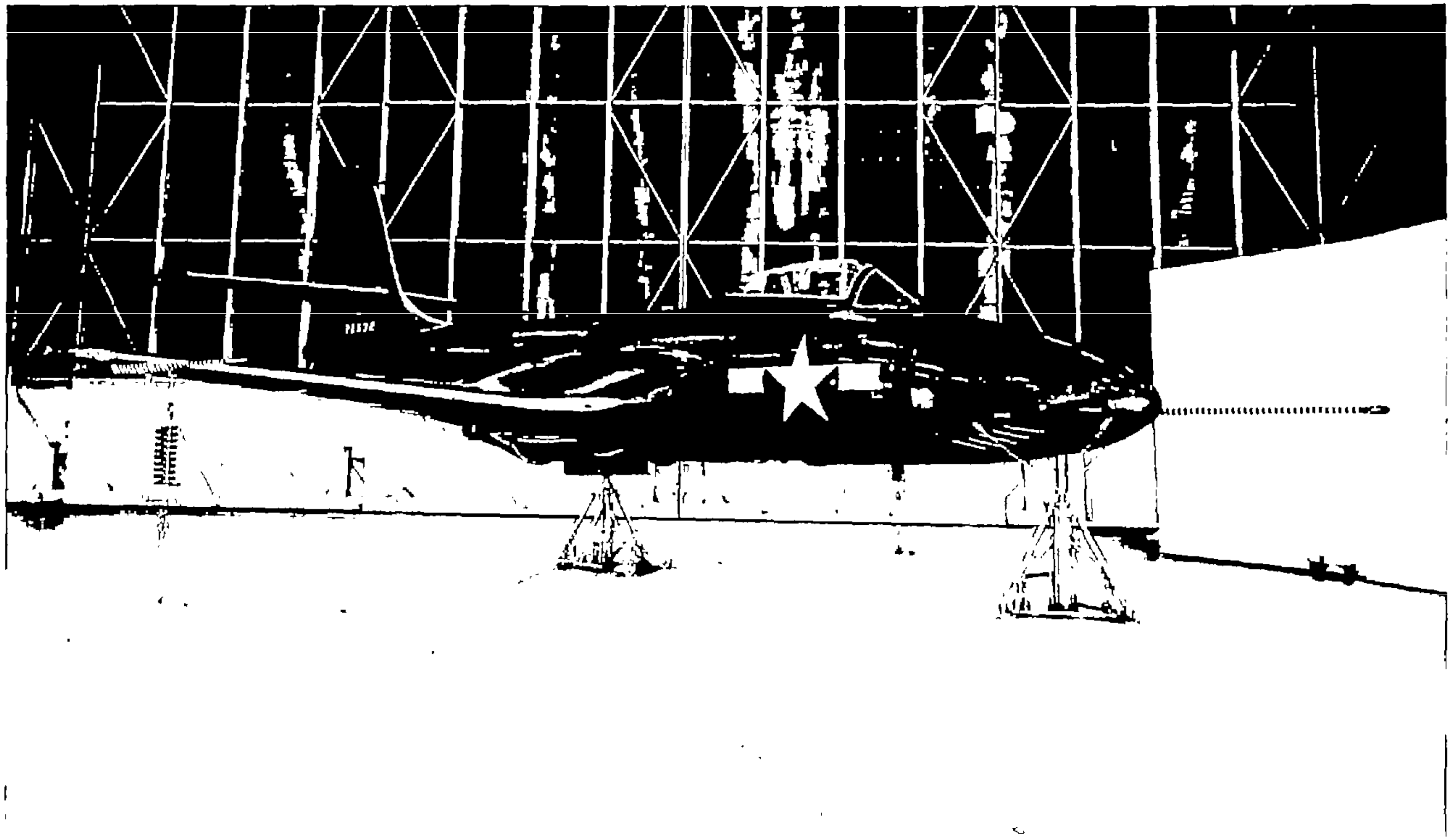


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Figure 6.- Test airplane as set up for determining the moment of inertia about the roll axis.

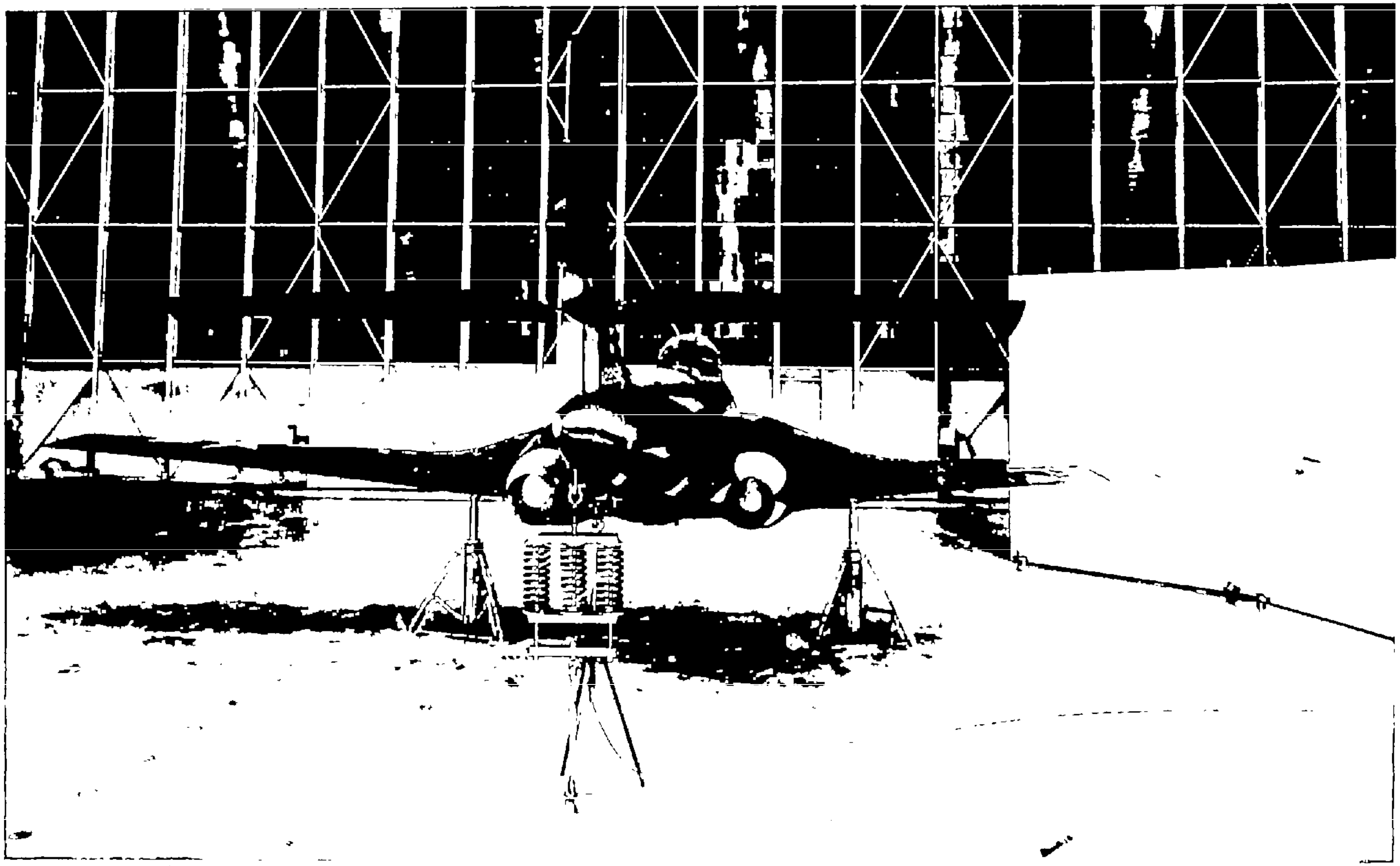

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Figure 7.- Test airplane as set up for determining the moment of inertia about the pitch axis.

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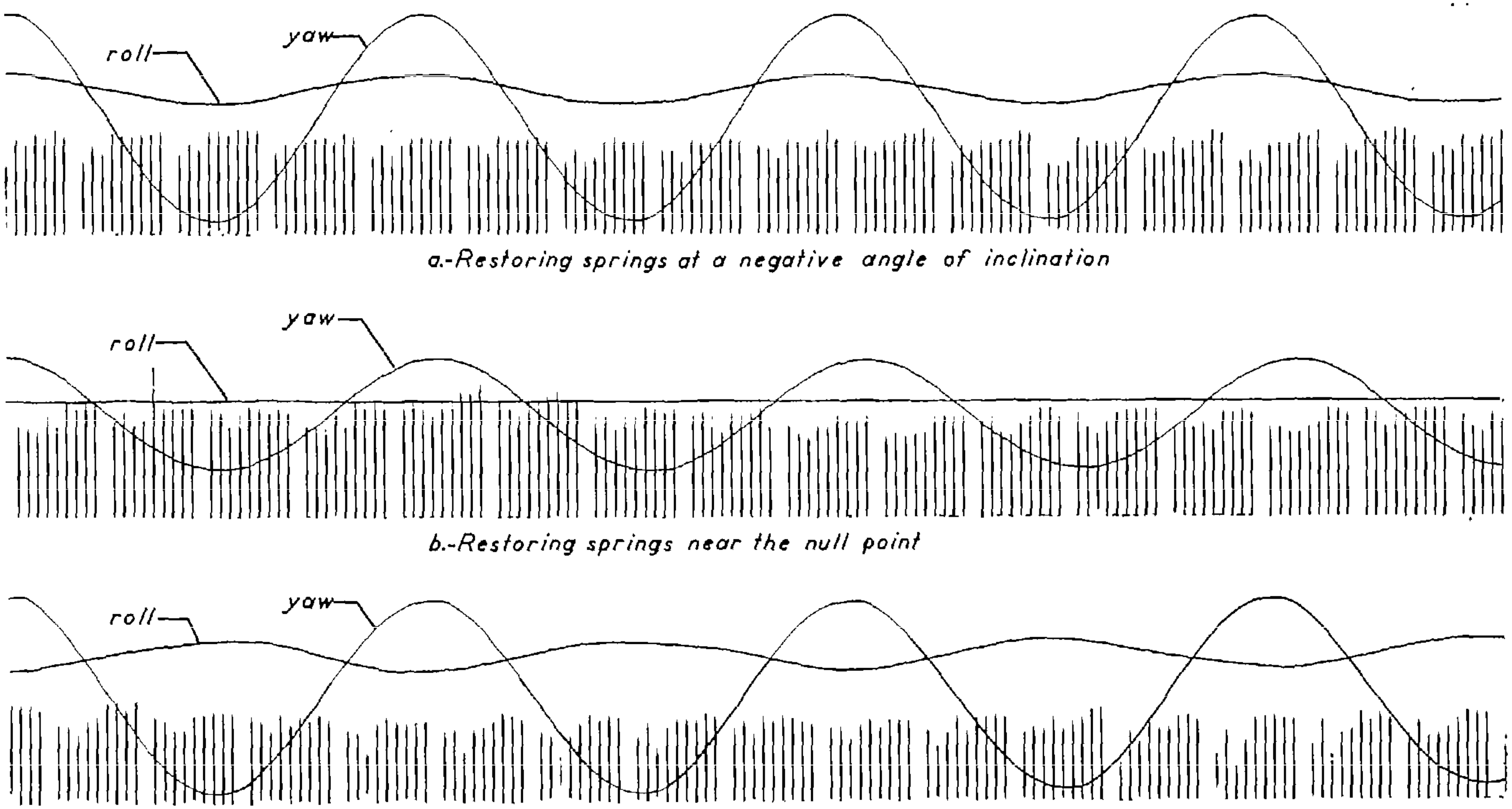
Figure 8.- Test airplane as set up for determining the moment of inertia about the yaw axis and the product of inertia.



a.-Restoring springs at a negative angle of inclination

b.-Restoring springs near the null point

c.-Restoring springs at a positive angle of inclination

Figure 9.- Typical time histories of yawing and rolling oscillations.

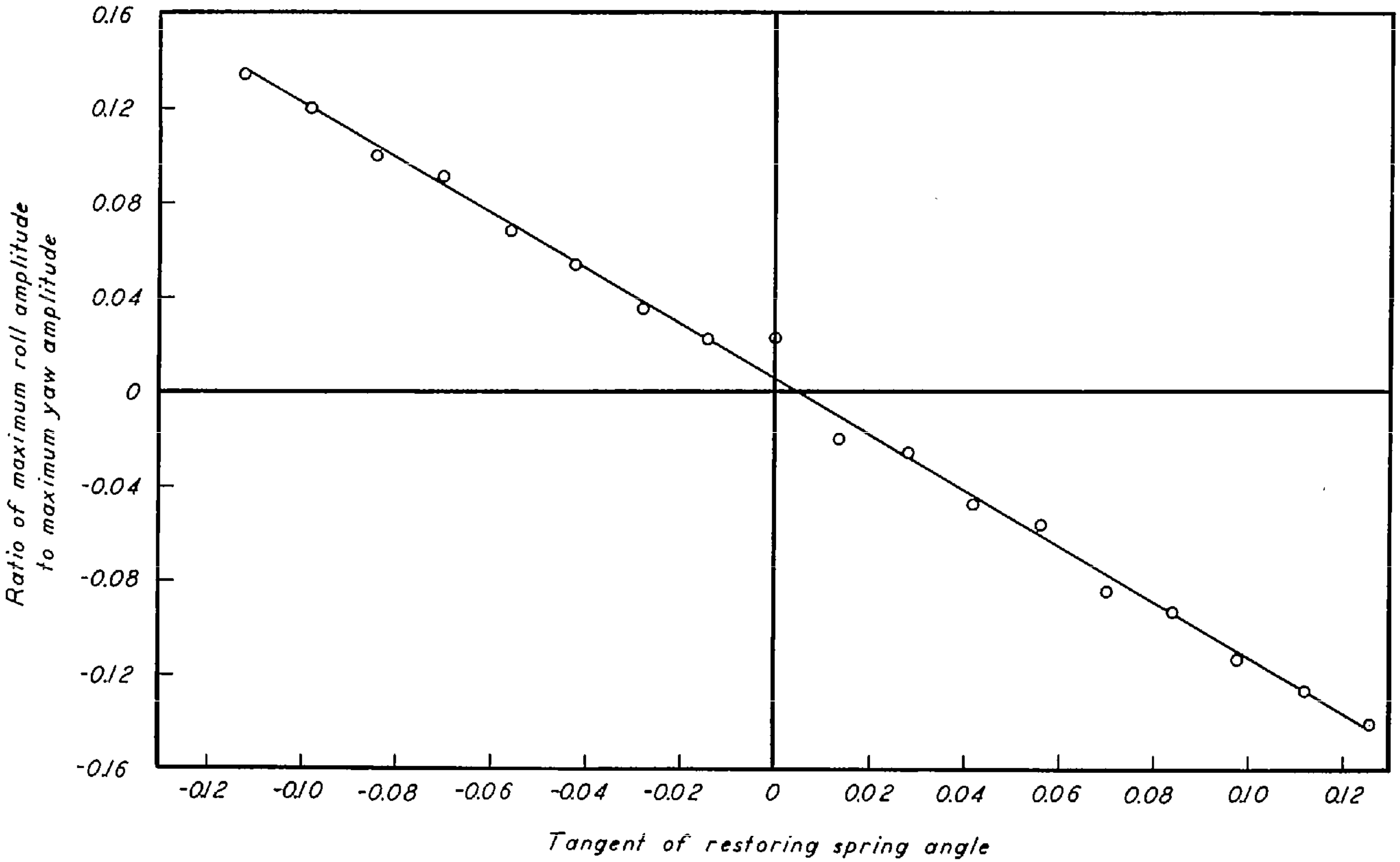


Figure 10.- Ratio of maximum roll amplitude to maximum yaw amplitude as a function of spring-restoring-moment direction for load condition 1 of the test airplane.

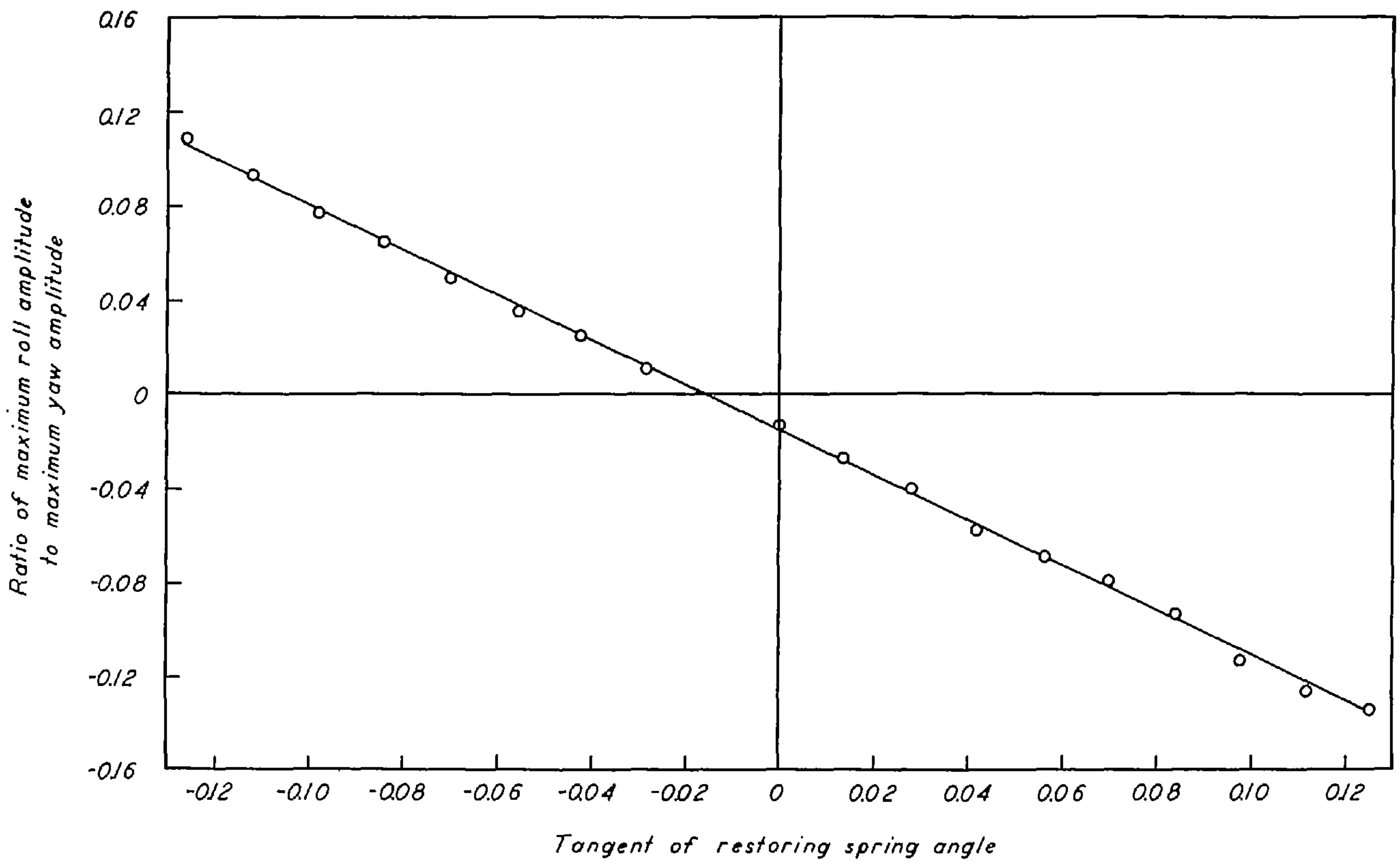


Figure 11.- Ratio of maximum roll amplitude to maximum yaw amplitude as a function of spring-restoring-moment direction for load condition 2 of the test airplane.

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